Written Exam Economics Winter 2018-19

Advanced Macroeconomics: Structural Vector Autoregressive Analysis

January 5, 10:00 — January 7, 10:00

This exam question consists of 6 pages in total.

Answers only in English. A take-home exam paper cannot exceed 15 pages plus an additional 10 pages with supplementary material – and one page is defined as 2400 keystrokes.

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!
Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Practical Information

Note the following formal requirements:

- This is an individual examination. You are not allowed to cooperate with other students or other people, see the focus on exam cheating above.
- The assignment consists of Sections 1-6 with 19 questions to be answered. Please answer all questions.
- The exam paper should not exceed 15 pages. A maximum of 10 pages of supporting material (graphs, estimation output, etc.) can accompany the paper as appendices. You may refer to the computer output in the appendices when answering the questions or you can include tables and/or graphs in the main text. Also, you may add clarifying comments in the output as part of your answer. It is not necessary to provide the MATLAB code you are using, unless otherwise stated in the question.
- All pages must be numbered consecutively and marked with your exam number. **You should not write your name on the exam paper.**
- Your paper must be uploaded on the course page in Absalon at the given time. The exam paper (including supporting material) must be in PDF-format and collected in one file only; the uploaded file must be named 1234.pdf, where 1234 is your exam number.

Regarding the data for the exam paper, please note the following:

- All assignments are based on different data sets. You should use the data set (monthly data covering the period 1974:1-2017:12) located in the MATLAB file 1234.mat, where 1234 is your exam number. This MATLAB file contains the data (y), the dates (dates) and the name of the variables (names). You can load this file into MATLAB directly using 'load 1234.mat'.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

Background

The topic for this project examination is the Purchasing Power Parity theorem stating that the foreign exchange rate adjusts to changes in relative prices in two countries. The purpose of the examination is to assess your understanding of structural vector autoregressive (VAR) models. Substantial emphasis will be placed on using your programming skills in MATLAB. Specifically, the examination assesses theoretical and practical knowledge of structural vector autoregressive models within stationary and non-stationary frameworks including assessing empirical results, using different approaches to identify VAR models and be able to use MATLAB to generate empirical results. You can use any MATLAB functions that you have programmed yourself or any functions uploaded to Absalon during the course except when otherwise stated. You are not allowed to use other programs or built-in MATLAB functions except for those that are specified in the questions below. The assignment requires some additional coding.

Most questions in the examination are applied, concerning the empirical example outlined below. When you answer these empirical questions, please explain and motivate your answers as detailed as possible, preferably with reference to the underlying theory.

Purchasing Power Parity (PPP) states that

$$S_{12} = \frac{P_1}{P_2} \tag{1}$$

where S_{12} is the nominal exchange rate between countries 1 and 2, P_1 is the price level in country 1 and P_2 is the price level in country 2. The real exchange rate is defined as

$$\frac{P_1}{P_2 S_{12}} \tag{2}$$

implying that the expected real exchange rate is equal to unity. If we take the log of the PPP relation in equation (1) and rearrange we obtain

$$\ln P_1 - \ln P_2 - \ln S_{12} = 0. \tag{3}$$

If PPP holds and prices and exchange rates are $I(1)$-processes, then the three variables must be cointegrated with cointegration vector

$$\begin{bmatrix} 1 & -1 & -1 \end{bmatrix} \tag{4}$$

The assignment will guide you through an empirical analysis of PPP including estimation and analysis of the cointegrated VAR model, identification of the structural cointegrated VAR model and robustness analysis.

1. The data is already in natural logarithms. Construct the real exchange rate and plot the data including the relative price ($\ln P_1 - \ln P_2$) and perform graphical analysis in order to assess the degree of integration of all five variables (the nominal exchange rate, the two price levels, the relative price and the real exchange rate).

The Vector Error Correction Model

Suppose that all three variables in the PPP relation are either $I(1)$ or $I(0)$ and that the underlying data generating process is a 3-dimensional Vector Autoregressive (VAR) model,

$$y_t = \nu + A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t \tag{5}$$

where $y_t = \begin{bmatrix} \ln P_1 & \ln P_2 & \ln S_{12} \end{bmatrix}'$, p is the lag length, ν is a constant term and u_t is a 3-dimensional zero mean white noise process with covariance matrix Σ_u such that $u_t \sim (0, \Sigma_u)$. Then we can rewrite the VAR model as the following Vector Error Correction (VEC) model

$$\Delta y_t = \nu + \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t \tag{6}$$

where

$$\Pi = -\left(I_3 - A_1 - \ldots - A_p\right)$$

and

$$\Gamma_i = -\left(A_{i+1} + \cdots + A_p\right) \quad \text{for } i = 1, \ldots, p-1.$$

The rank of Π is equal to the number of cointegration vectors r and can be decomposed as a product of two $3 \times r$ matrices of full rank, $\Pi = \alpha\beta'$ where α is the $3 \times r$ adjustment coefficients and β is the $3 \times r$ cointegration vectors.

2. Formulate a well-specified VEC model for y_t similar to the VEC model above. Explain your workflow and how you argue for your choice of the number of autoregressive lags.

3. Test for multivariate autocorrelation, heteroscedasticity and normality. Does you model satisfy the underlying assumptions?

Testing for cointegration

4. For your preferred model, proceed by testing for cointegration using the MATLAB function jcitest. Explain your approach and how you proceed to find the number of cointegration vectors in the system, that is the rank r. Do you use different sources of information when determining the rank? If so, explain how you argue.

5. Discuss and argue for the preferred way to include deterministic components in the model.

6. Impose your preferred rank and test hypotheses on the cointegration space using the MATLAB function jcontest. Start with tests for exclusion, stationarity and weak exogeneity. Explain the meanings of these tests.

7. Test the null hypothesis that PPP holds using the MATLAB function jcontest. Explain how this test relates to the exclusion and stationarity tests.

8. Split the sample into two equal sized sub-samples and perform tests for exclusion, stationarity, weak exogeneity and the null hypothesis that PPP holds. Comment on the importance of the sample length for these tests.

9. Impose $r = 1$ and the theoretical cointegration vector (the assumption that PPP holds) and re-estimate the VEC model using the full sample and using your preferred lag length found above. Comment on the driving forces of the data in this model.

Identification of structural model

10. What are the main differences between short- and long-run restrictions. What are the main assumptions underlying these two alternatives to identify structural VAR models? You are not required to discuss over-identification or non-recursive identification.

11. Write down the reduced form and structural form Common Trends model consistent with the VEC model. Show how these two representations are related. Provide an interpretation of the three structural shocks in the structural model. If you cannot provide names for these shocks, try to explain how they affect the data and the real exchange rate under the maintained assumptions.

12. What is the consequence for the long-run multiplier if we assume that PPP holds?

13. Use the estimated reduced form VEC model together with identifying restrictions (a combination of short- and long-run restrictions) to identify the three structural shocks. Show how these shocks can be identified including the number of long-run and short-run restrictions needed to just identify the structural shocks. Consider only null restrictions.

14. Outline how the MATLAB solver can be used to impose these restrictions.

Impulse responses and forecast error variances

15. Use the MATLAB solver to identify the structural model. Check that the solver provides a valid identification and compute the variance-covariance matrix of the identified structural shocks. Please, provide the MATLAB code you are using to identify the shocks in the appendix. It must include a description of the null restrictions you impose.

    If you fail computing the B_0^{-1} matrix above, please use the restrictions.p file. This file works as a standard m-file but the coding is concealed and there is no way to convert the p-file into an m-file. Note that the restrictions.p file is set up to use the MATLAB solver to compute the B_0^{-1} matrix using a generic identification based on estimates from the VEC model. You can use the standard coding to initiate the solver. To do this, you need to include the following code into your MATLAB m-file. Note: Make sure that you don't have any restrictions.m files in the same folder and that the function vec.m is in the same folder. The same file can be used in a bootstrap.

```
% Set global variables
global SIGMA GAMMA alpha beta alpha_perp beta_perp Xi p;
% Set options for fsolve
warning off
options=optimset('TolX',1e-10,'TolFun',1e-10,'MaxFunEvals',1e+10,'MaxIter',2000);
```

```
% Compute B0inv
B0inv=fsolve('restrictions',randn(K,K),options);
```

16. Estimate the structural VAR model and compute impulse response functions (with bootstrap confidence bands using the delta method, i.e., the standard residual based recursive design bootstrap with intervals based on bootstrap standard errors) and variance decompositions (with bootstrap standard errors using Efron's percentile intervals). In addition, compute the implied impulse responses of the real exchange rate to all shocks and the corresponding forecast error variance decomposition. You can show forecast error variance decompositions in either a table or in a graph. Interpret your results.

17. Show a plot of the accumulated permanent shock(s) and the three variables. Discuss the results. Then plot the real exchange rate together with the accumulated permanent shock(s). Is it possible to draw conclusions regarding the driving forces of the variables?

Extensions

18. Re-estimate the VEC model using the estimated cointegration vector instead of the theoretical one. Use the same number of lags as above and compute impulse response functions (with bootstrap confidence bands using the delta method). Compare these to the impulse response functions you obtained when imposing the PPP condition. How sensitive are impulse responses to deviations from PPP? Compute the impulse responses of the real exchange rate. Interpret and compare to the case when PPP holds. As above, in case you have not managed to use the solver, you can use the p-file restrictions.p to perform these computations.

19. An alternative to using the MATLAB solver to compute the B_0^{-1} matrix is to use the approach suggested by Warne (1993). Outline this approach and show that the Warne approach yields a B_0^{-1} matrix identical to the one found by the solver. You can either work with the theoretical cointegration vector assuming that PPP holds or the estimated cointegration vector. Please, provide the code you are using in the appendix.